Equivalent: Jurnal Ilmiah Sosial Teknik
Volume 8, Issue 6, 2444-2459
e_ISSN: 2775-0833
https://jurnalequivalent.id/index.php/jequi/index
DOI: doi.org/10.59261/jequi.v8i6.461

Maritime Navigation Learning with Problem-Based Learning in Spherical Trigonometry

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Article Info:

Article history:
Received: September 08, 2026
Revised: November 09, 2026
Accepted: November 11, 2026
Available Online: November 17, 2026

Keywords:
Problem-Based Learning;
Spherical Trigonometry;
Great Circle Navigation;
Nautical Cadets;
Learning Outcomes;

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Abstract

Background: Spherical trigonometry is essential for maritime navigation; however, conventional instruction often emphasizes procedural drills without connecting mathematical formulas to authentic bridge operations.
Objective: This study examined changes in nautical cadets' learning outcomes and perceived learning experiences following a contextualized Problem-Based Learning (PBL) intervention in spherical trigonometry.
Methods: A quantitative quasi-experimental study using a One-Group Pretest–Posttest Design was conducted with 65 nautical cadets at Politeknik Pelayaran Sorong during the 2025/2026 academic year. The PBL intervention integrated authentic Great Circle navigation problems, manual calculations using the Spherical Law of Cosines, and digital cross-checking. Data were collected using pretest and posttest instruments, navigation task evaluations, and a 50-item perception questionnaire. The data were analyzed using descriptive statistics, the Wilcoxon signed-rank test, Modified Normalized Gain (N-Gain), and effect size (image).
Results: Learning outcomes increased significantly, with the mean score rising from 59.45 (SD = 19.95) to 71.32 (SD = 22.18) (image), indicating moderate improvement (Modified N-Gain = 0.340) and a moderate-to-large effect size (image). Cadets demonstrated high computational consistency between manual and digital methods, with differences ranging from 0.000% to 0.050%. They also reported positive perceptions of conceptual understanding, motivation, self-directed learning, and collaboration, despite a slight descriptive decline in perceived problem-solving confidence.
Conclusion: Contextualized PBL was associated with improved learning outcomes and high computational consistency. Given the single-group design, these findings indicate changes associated with the intervention rather than establishing definitive causality. Controlled, multi-institutional studies are recommended.

To cite this article: Yudhiyono, F., & Muhammad, M. I. (2026). Maritime navigation learning with problem-based learning in spherical trigonometry. *Equivalent: Jurnal Ilmiah Sosial Teknik*, 8(6), 2444-2459. https://doi.org/10.59261/jequi.v8i6.461

INTRODUCTION

Maritime shipping plays a central role in international trade, accounting for approximately 80% of global merchandise trade by volume (IMO, 2023; UNCTAD, 2021). This central role makes effective navigation planning one of the essential competencies required of future maritime officers. Long-distance maritime navigation requires mathematical methods that account for the Earth's approximately spherical geometry. Within this mathematical framework, great-circle navigation and the Spherical Law of Cosines can be used to determine the surface distance between two geographical positions (Bowditch, 2017). These mathematical procedures are directly relevant to practical navigation tasks, including voyage distance calculations and route planning. For nautical cadets, spherical trigonometry is therefore more than a course

requirement; it provides a mathematical foundation for calculating, interpreting, and independently verifying navigation information in professional settings.

Contemporary mathematics education increasingly seeks to place learners in situations where mathematical ideas must be interpreted and applied rather than treated solely as procedures to be reproduced (Santos-Trigo, 2024). When learners engage with contextualized or nonroutine problems, mathematical procedures can be connected to situations that require the interpretation, evaluation, and comparison of solutions (Suryanti et al., 2022). Evidence from mathematics education also demonstrates the potential of STEM-oriented learning to support important learner outcomes. For example, Ilyas et al. (2022) reported improvements in students' critical thinking and learning motivation following STEM-based mathematics instruction. In higher education, PBL is grounded in constructive, self-directed, collaborative, and contextual learning processes in which learners engage with problems rather than acquire knowledge solely through direct instruction (Savery, 2015). PBL further positions problem-solving activities as the starting point of learning: learners work collaboratively to identify what they need to know, independently pursue relevant knowledge, apply it to the problem, and reflect on the problem-solving process (Hmelo-Silver, 2004).

A systematic review by Loyens et al. (2023) further indicates that problem- and project-based learning environments can support higher-order and critical thinking through engagement with problems and collaborative learning processes. This finding is consistent with recent systematic evidence on HOTS-oriented PBL interventions in mathematics (Aba-Oli et al., 2025). Furthermore, problem-based learning has been shown to significantly enhance student motivation and engagement across diverse educational settings (Wijnia et al., 2024). Taken together, these perspectives support the use of PBL to reposition spherical trigonometry from a predominantly procedural topic to a contextualized learning experience in which nautical cadets can reason mathematically, address authentic navigation problems, and relate mathematical concepts to professional maritime practice.

Although PBL has been widely discussed in mathematics education, fewer empirical studies have examined how it can be structured around discipline-specific mathematical tasks in maritime education. Previous research has connected PBL, trigonometry, and spherical triangle concepts with ship navigation problems, demonstrating the potential of contextualized mathematics learning in maritime settings (Sriantini & Abadi, 2024). However, the cited studies do not clearly examine a structured maritime mathematics PBL sequence in which authentic great-circle navigation problems are used both to develop spherical trigonometric reasoning and to verify digitally generated navigation outputs through manual computation. A related research gap concerns the pedagogical integration of digital navigation technologies with mathematical verification.

Recent research in maritime education has demonstrated the use of digital navigation simulations incorporating spherical trigonometry and great-circle route modeling. However, its primary emphasis is on navigation technology and operational training rather than manual mathematical verification (Feng et al., 2026). In parallel, Baric et al. (2021) examined the mathematical and computational reliability of great-circle navigation and highlighted the continuing importance of performing navigation calculations without electronic aids. However, their work addresses the navigational computation problem rather than the use of great-circle calculations as a structured mathematical learning activity in which cadets compare manual calculations with digital outputs. The present study addresses these gaps by integrating PBL with authentic great-circle navigation problems and combining manual calculations based on the Spherical Law of Cosines with digital navigation tools to examine cadets' mathematical learning, verification skills, and navigation-related reasoning.

The increasing use of digital and automated technologies in maritime navigation has shifted navigators' roles from performing certain navigation tasks manually toward supervising, interpreting, and validating information produced by technological systems. Technologies such as Electronic Chart Display and Information Systems (ECDIS), Global Navigation Satellite Systems (GNSS), and integrated navigation systems provide increasingly sophisticated support for navigation and decision-making. However, greater automation also creates new requirements for human supervision and control. Veitch & Andreas Alsos (2022), in a systematic review of human–

AI interaction in autonomous ship systems, emphasized that increasing automation does not eliminate the need for human control; instead, it introduces new challenges concerning how humans supervise and interact with automated navigation systems.

A systematic review by Crestelo Moreno et al. (2022) likewise demonstrates the continuing importance of human factors in maritime safety, particularly in relation to the training, experience, performance, and working conditions of vessel traffic service operators. These findings reinforce the importance of maintaining navigators' ability to independently evaluate information generated by digital navigation systems. Research on ECDIS has similarly emphasized the importance of position validation and cross-checking to reduce overreliance on electronically displayed navigation information. In this context, manual mathematical calculation can serve not only as a means of solving navigation problems but also as an educational mechanism through which cadets learn to verify, interpret, and critically evaluate digitally generated navigation information.

IMO guidance recognizes ECDIS as a safety-relevant navigation system whose effective use requires appropriate training, an understanding of system limitations, and proper operational practices (IMO, 2017). Rather than eliminating the navigator's role, electronic navigation systems place greater emphasis on navigators' ability to interpret and evaluate information produced by technological systems. This concern is reinforced by human–automation research. Endsley (2017) demonstrated that increasing levels of automation can create challenges for human situation awareness and operators' ability to intervene effectively when automated systems require human control. Similarly, Parasuraman & Riley (1997) identified overreliance on automation as a form of automation misuse that can contribute to monitoring failures and biased decision-making. In maritime navigation, Bowditch (2017) likewise emphasizes that modern navigators must understand the capabilities and limitations of electronic navigation systems and evaluate the accuracy of their outputs while retaining sufficient conventional navigation skills to respond when electronic systems are unavailable or unreliable. Together, these perspectives indicate that maritime mathematics instruction should combine digital proficiency with independent mathematical verification. In this study, manual calculation is therefore used as a learning activity for checking the mathematical plausibility of digital navigation outputs rather than as a substitute for electronic navigation systems.

Previous research has examined the use of Problem-Based Learning (PBL) in mathematics teacher education. For example, Bosica et al. (2021) investigated PBL within a secondary mathematics preservice teacher education course and reported changes in prospective teachers' beliefs, orientations, and teaching efficacy. Building on this line of research, the present study extends PBL to a discipline-specific maritime mathematics context through the integration of three complementary components: PBL pedagogy, spherical trigonometry taught through authentic great-circle navigation problems, and manual verification of digitally generated navigation results using the Spherical Law of Cosines. This integrated approach is consistent with the need to support the transfer of mathematical knowledge beyond the immediate learning situation by connecting mathematical concepts with contexts in which they can be meaningfully applied (Orón & Lizasoain, 2023). It also complements recent developments in maritime education that integrate digital navigation technologies with foundational navigation knowledge to improve students' operational capabilities (Feng et al., 2026). By combining authentic navigation problems, mathematical reasoning, manual computation, and digital verification within a PBL framework, the present study provides a contextualized mathematics learning approach in which nautical cadets not only apply spherical trigonometry to professional maritime problems but also independently compare and evaluate manually calculated results against digitally generated navigation information.

Accordingly, this study aims to examine changes in nautical cadets' spherical trigonometry learning outcomes following a contextualized Problem-Based Learning (PBL) intervention using authentic great-circle navigation problems. The study focuses on three interconnected aspects: the application of the Spherical Law of Cosines to great-circle distance calculations, the comparison of manually calculated navigation results with digitally generated outputs, and cadets' perceptions of the contextualized PBL learning experience. By integrating mathematical reasoning, authentic maritime problems, and digital navigation tools, the study seeks to determine

whether PBL can help cadets apply spherical trigonometric concepts beyond procedural calculations and use mathematical knowledge to interpret and evaluate navigation information. This focus responds to continuing interest in problem-based mathematics learning while extending the evidence base to a discipline-specific professional education context that remains underrepresented in the literature. Recent systematic evidence confirms the broad use of PBL in mathematics education while also highlighting variation in learning contexts, instructional designs, and outcomes across studies (Ayari et al., 2025).

The study therefore contributes to the literature by examining PBL not merely as a general instructional strategy but as a contextualized approach in which mathematical concepts, professional navigation problems, and digital technologies are deliberately connected. The study also examines whether manual mathematical computation can function as a verification activity through which cadets critically evaluate digitally generated navigation information. Based on this focus, the study addresses three interrelated research questions: the extent to which cadets' learning outcomes in spherical trigonometry change following the implementation of Problem-Based Learning through authentic great-circle navigation problems (RQ1); the extent to which cadets' manual great-circle distance calculations using the Spherical Law of Cosines correspond to digitally generated navigation results (RQ2); and how cadets perceive the contextualized PBL learning experience in terms of conceptual understanding, learning motivation, problem-solving skills, self-directed learning, and collaboration and communication during authentic maritime navigation activities (RQ3).

METHOD

This study employed a quantitative quasi-experimental approach using a one-group pretest–posttest design to examine changes in cadets' learning outcomes following the implementation of Problem-Based Learning (PBL) in spherical trigonometry (Anderson-Cook, 2005; Creswell & Creswell, 2017; Fraenkel & Wallen, 1990). The same participants completed a pretest before the intervention and a posttest afterward, with their scores compared to assess changes in learning outcomes. The study was conducted at Politeknik Pelayaran Sorong, Indonesia, during the first semester of the 2025/2026 academic year (September 2025–February 2026) and involved 65 nautical cadets enrolled in the Applied Mathematics course.

Participants were purposively selected based on predefined instructional criteria, including completion of the prerequisite navigation mathematics course and no prior formal instruction in spherical trigonometry. The PBL intervention was integrated into the spherical trigonometry topic and involved authentic great-circle navigation problems, manual calculations using the spherical law of cosines, and comparisons with digitally generated navigation results. The same group participated in the pretest, intervention, posttest, and perception questionnaire. Because no control group was included, the observed changes were interpreted as changes following the intervention rather than as definitive causal effects attributable exclusively to PBL (Shadish & Cook, 2002).

The research procedure consisted of four stages: pre-intervention assessment, PBL intervention, post-intervention assessment, and data analysis. First, cadets completed a pretest to assess their initial learning outcomes in spherical trigonometry and their ability to apply relevant concepts to navigation-related problems. Second, the cadets participated in a contextualized PBL intervention based on authentic maritime navigation problems, requiring them to identify relevant mathematical concepts, formulate solution strategies, perform manual great-circle calculations using the spherical law of cosines, and compare their results with digitally generated navigation outputs. Third, following the intervention, cadets completed a posttest aligned with the same learning objectives and a comparable level of difficulty as the pretest, followed by a structured questionnaire addressing conceptual understanding, learning motivation, problem-solving skills, self-directed learning, and collaboration and communication. Finally, the collected data were analyzed using descriptive and inferential procedures. Pretest and posttest scores were examined to assess changes in learning outcomes, questionnaire responses were analyzed descriptively, and the great-circle task was evaluated by comparing manual calculations with corresponding digitally generated navigation results.

The PBL intervention was adapted from the framework proposed by Savery (2015) and

contextualized to the professional learning needs of nautical cadets. The intervention centered on an authentic great-circle navigation problem involving representative routes (e.g., Tokyo, Jakarta, and Dubai), enabling cadets to connect spherical trigonometry with practical maritime navigation (Sriantini & Abadi, 2024). The learning process consisted of five interconnected activities: problem orientation, problem analysis and planning, manual mathematical calculation, digital cross-checking, and presentation and reflection, as summarized in Table 1.

Table 1
PBL Intervention Stages and Learning Activities

PBL Stage	Cadet Activity	Learning Output
Problem orientation	Identify the navigation problem from an authentic Great Circle scenario	Problem formulation
Problem analysis and planning	Identify variables and relevant spherical-trigonometric principles	Mathematical model
Manual calculation	Apply the Spherical Law of Cosines and related calculations	Manual navigation result
Digital cross-checking	Compare manual and digitally generated results	Result comparison
Presentation and reflection	Explain solutions and interpret differences	Conclusion and reflection

During problem orientation, cadets were presented with the geographical coordinates of the selected locations and guided to identify the information required to determine great-circle distances. During problem analysis and planning, cadets worked collaboratively in small groups to identify known and unknown variables, interpret latitude and longitude, formulate the corresponding spherical-trigonometric representation, and select appropriate solution strategies. The instructor acted as a facilitator by providing guiding questions, scaffolding, and clarification rather than directly presenting the solution. During the calculation stage, cadets represented the navigation problem as a spherical triangle and calculated the great-circle distance manually using the spherical law of cosines (Sinnott, 1984):

$$\cos(c) = \cos(a)\cos(b) + \sin(a)\sin(b)\cos(C) \quad (1)$$

where a, b, and c represent the sides of the spherical triangle (expressed in angular distance) and C is the included angle (difference in longitude). Cadets interpreted the geographical coordinates mathematically, substituted the relevant values, performed the calculations, and interpreted the resulting distance in the context of maritime navigation. Where required by the learning task, cadets also determined the spherical excess using:

$$E = A + B + C - 180° \quad (2)$$

where A, B, and C are the interior angles of the spherical triangle. The spherical excess was converted to radians (E_rad) before the spherical-triangle area was calculated using:

$$\text{Area} = E_{rad} R^2 \quad (3)$$

where R represents the Earth's mean radius (R ≈ 6,371 km). These calculations provided additional opportunities to apply spherical-trigonometric relationships and interpret the geometric properties of the navigation problem. In the digital cross-checking stage, cadets compared their manually calculated Great Circle distances with corresponding digitally generated navigation results and discussed possible differences; this comparison was made against results generated using the Movable Type Scripts Great Circle Calculator, used as a computational reference rather than as an independent measure of navigational competence, which enabled cadets to identify discrepancies between manual and digital results and discuss possible mathematical or computational sources of differences, thereby reinforcing the importance of critically evaluating digitally generated navigation information (Baric et al., 2021). Computational consistency was assessed using the percentage difference between the manual and digital results:

$$\text{Percentage Difference} = ((D_m - D_d) / D_d) \times 100\% \quad (4)$$

where D_m represents the manually calculated distance and D_d represents the digitally generated distance. The differences were interpreted descriptively to determine the consistency

between the two calculation methods. This activity connected spherical-trigonometric reasoning with technology-supported navigation and emphasized the independent mathematical verification of digital navigation outputs. The activity concluded with group presentations and reflection, during which each group presented its problem-solving procedures, calculations, results, and navigational interpretations to the class.

Cadets explained the transformation of geographical coordinates into spherical-trigonometric variables, the application of the spherical law of cosines, and the consistency between manual and digital results. Peers were encouraged to ask questions, evaluate alternative solution strategies, and identify possible computational errors. The presentations were followed by a structured reflection in which cadets discussed the mathematical concepts applied, difficulties encountered, possible reasons for discrepancies between manual and digital results, and the relevance of spherical trigonometry to professional maritime navigation. These activities encouraged cadets to articulate mathematical reasoning, evaluate solution strategies, communicate ideas, and connect classroom learning with authentic navigation practice (Savery, 2015).

Two instruments and one performance-based learning task were used in the study (Hill et al., 2022). The pretest and posttest consisted of equivalent essay and computational items designed to assess cadets' learning outcomes in spherical trigonometry, focusing on conceptual understanding and the application of spherical-trigonometric principles to navigation-related problems. The items were developed in accordance with the learning outcomes and instructional content of the spherical trigonometry component of the Applied Mathematics course. The pretest was administered before the PBL intervention and the posttest after its completion, with learning improvement assessed by comparing the scores of the same cadets.

Given the one-group pretest–posttest design, this comparison was interpreted as evidence of changes following the intervention rather than as evidence of superiority over a control group. To address RQ2, the authentic great-circle navigation task was assessed across four aspects: interpretation of geographical coordinates, appropriate application of the spherical law of cosines, accuracy of manually calculated great-circle distances, and consistency between manual and digitally generated results. Cadets were required to transform geographical coordinates into spherical-trigonometric variables, select and apply the appropriate formula, calculate great-circle distances manually, and compare their results with a digital reference.

The percentage difference between manual and digital calculations was used as an indicator of computational consistency, and the task was treated as complementary evidence rather than as an independent experimental outcome comparable to the pretest and posttest. Finally, cadets' perceptions were measured using a five-point Likert-scale questionnaire consisting of 50 items covering two instructional conditions, Conventional Learning and Problem-Based Learning (Mohd Rokeman, 2024). The questionnaire comprised five dimensions—conceptual understanding, learning motivation, problem-solving skills, self-directed learning, and collaboration and communication—with five items for each dimension under each instructional condition, resulting in 25 items for Conventional Learning and 25 items for PBL. The pretest and posttest were designed to cover the same learning outcomes and content domains.

Because the available study record does not report a separate statistical equivalence coefficient or retain an item-by-item blueprint, instrument equivalence was assessed based on parallel content coverage rather than an independent equivalence statistic. The available study record also does not preserve a separate administration schedule for the two questionnaire condition blocks. Consequently, the comparison between the Conventional Learning and PBL questionnaire responses was treated as descriptive self-report evidence rather than as an independently controlled pre–post measure.

The questionnaire was evaluated using corrected item–total correlations and Cronbach's alpha. Corrected item–total correlations ranged from 0.319 to 0.787 for the Conventional Learning items and from 0.539 to 0.866 for the PBL items, with all coefficients exceeding the minimum criterion of 0.30 (Field, 2018). Cronbach's alpha ranged from 0.712 to 0.856 for the Conventional Learning dimensions and from 0.870 to 0.915 for the PBL dimensions, indicating acceptable to excellent internal consistency Field (2018), as detailed in Table 2.

Table 2
Item Validity and Internal Consistency of the Conventional Learning and PBL Questionnaires

Learning Condition	Dimension	Items	Cronbach's α	Corrected Item–Total Correlation	Reliability
Conventional	Conceptual Understanding	5	0.800	0.319–0.744	Good
Conventional	Learning Motivation	5	0.712	0.410–0.534	Acceptable
Conventional	Problem-Solving Skills	5	0.741	0.444–0.559	Acceptable
Conventional	Self-Directed Learning	5	0.786	0.412–0.679	Acceptable
Conventional	Collaboration & Communication	5	0.856	0.554–0.787	Good
PBL	Conceptual Understanding	5	0.889	0.702–0.771	Good
PBL	Learning Motivation	5	0.913	0.690–0.866	Excellent
PBL	Problem-Solving Skills	5	0.915	0.757–0.825	Excellent
PBL	Self-Directed Learning	5	0.890	0.539–0.859	Good
PBL	Collaboration & Communication	5	0.870	0.652–0.806	Good

Overall, the corrected item–total correlations and Cronbach's alpha coefficients indicated satisfactory item-level performance and internal consistency for both instructional conditions. All quantitative data were analyzed using IBM SPSS Statistics version 27 (Field, 2018). Descriptive statistics, including the mean, standard deviation, minimum, maximum, frequency, and percentage, were calculated to summarize the pretest, posttest, and questionnaire data. The normality of the pretest and posttest scores was assessed using the Shapiro–Wilk test. Because the learning-outcome data did not meet the normality assumption, differences between pretest and posttest scores were examined using the two-tailed Wilcoxon signed-rank test at a significance level of $\alpha=0.05$ (Field, 2018). The effect size of the Wilcoxon test was calculated using:

$$r = |Z| / \sqrt{N} \quad (5)$$

where Z represents the standardized test statistic and (N) represents the total number of paired observations (Field, 2018). Learning improvement was additionally evaluated using the Modified Normalized Gain (Modified N-Gain) to describe the magnitude of improvement relative to cadets' initial performance. For the authentic great-circle navigation task, manual and digitally generated distances were compared separately for each route. In addition to descriptive comparisons, the absolute difference and percentage difference between manual and digital distances were calculated to describe numerical consistency between the two calculation methods. For the perception questionnaire, mean scores, standard deviations, frequencies, and percentages were calculated for each of the five dimensions under the Conventional Learning and PBL conditions. The analysis was used descriptively to characterize cadets' perceptions across the five dimensions. The alignment between the research questions, data sources, indicators, and analytical procedures is summarized in Table 3. The gain index was calculated based on the increase from pretest to posttest relative to the remaining attainable score on the 0–100 scale, and the reported value was interpreted descriptively alongside the Wilcoxon test and effect size.

Table 3
Alignment Between Research Questions, Data Sources, and Analysis

Research Question	Data Source	Main Indicator	Analysis
RQ1. To what extent do cadets' learning outcomes in spherical trigonometry improve following the implementation of PBL?	Pretest and posttest	Score changes	Descriptive statistics, Shapiro–Wilk, Wilcoxon Signed-Rank Test, Modified N-Gain, effect

			size
RQ2. How closely do cadets' manual Great Circle distance calculations correspond to digitally generated navigation outputs?	Authentic Great Circle navigation task	Manual distance, digital distance, percentage difference	Descriptive analysis, Percentage Difference
RQ3. How do cadets perceive the contextualized PBL learning experience?	50-item Likert-scale questionnaire	Five perception dimensions	Mean, frequency, percentage, descriptive comparison

RESULTS AND DISCUSSION

Results

Changes in Cadets' Learning Outcomes Following the PBL Intervention

The implementation of Problem-Based Learning (PBL) was followed by an improvement in cadets' learning outcomes in spherical trigonometry. As shown in Table 4, the mean score increased from 59.45 (SD = 19.95) in the pretest to 71.32 (SD = 22.18) in the posttest, representing an increase of 11.87 points. The minimum score remained unchanged at 16, whereas the maximum score increased from 88 to 96.

Table 4

Descriptive Statistics of Pretest and Posttest Scores

Assessment	N	Mean	SD	Min	Max
Pretest	65	59.45	19.95	16	88
Posttest	65	71.32	22.18	16	96

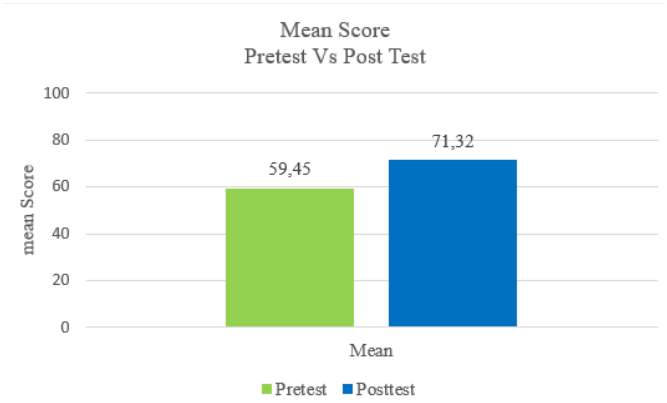


Figure 1. Comparison of Pretest and Posttest Scores

Because the learning-outcome data did not meet the assumption of normality, the Wilcoxon signed-rank test was used to examine the difference between pretest and posttest scores (Field, 2018). The test indicated a statistically significant difference between the two assessments ($Z = -3.890$, $p < .001$). The Modified N-Gain was 0.340, indicating a moderate level of learning improvement based on standard gain criteria. The Wilcoxon effect size was $r = 0.483$, indicating a moderate-to-large practical effect (Field, 2018). Overall, cadets demonstrated meaningful improvements in their spherical trigonometry learning outcomes following the PBL intervention, consistent with previous evidence on problem-based mathematics instruction (Ayari et al., 2025).

Application of the Spherical Law of Cosines in Great Circle Navigation and Digital Verification

To address RQ2, cadets' application of the spherical law of cosines was examined through authentic great-circle navigation tasks. Cadets transformed geographical coordinates into

spherical trigonometric variables, calculated great-circle distances manually, and subsequently compared their results with those generated by a web-based great-circle calculator. Across the three routes examined, the manual calculations produced distances that were remarkably close to the digital reference values. For the Harbor 1–Harbor 2 route, the mean manual distance was 2,818.91 km, compared with 2,819.07 km from the digital calculator, yielding a mean difference of 0.16 km (approximately 0.006%). For Harbor 2–Harbor 3, the corresponding values were 8,226.17 km and 8,230.23 km, respectively, with a difference of 4.06 km (approximately 0.049%). For Harbor 1–Harbor 3, the mean manual and digital distances were 8,442.90 km and 8,442.98 km, respectively, with a difference of 0.08 km (approximately 0.001%), as summarized in Table 5.

Table 5
Comparison of Manual and Digital Great Circle Calculations

Route	Manual Distance (km)	Digital Distance (km)	Difference (km)	Percentage Difference
Harbour 1–Harbour 2	2,818.91	2,819.07	0.16	0.005%
Harbour 2–Harbour 3	8,226.17	8,230.23	4.06	0.05%
Harbour 1–Harbour 3	8,442.90	8,442.98	0.08	≈0.00%

Overall, the percentage differences ranged from approximately 0.001% to 0.049%, indicating a very high degree of numerical consistency between the manual and digital calculations. These findings provide empirical evidence that cadets were able to apply the spherical law of cosines accurately to authentic great-circle navigation problems and effectively use digital outputs as computational cross-checks (Baric et al., 2021). The digital calculator was treated as an external computational reference rather than as an absolute gold standard of navigational accuracy.

Cadets' Perceptions of the Contextualized PBL Learning Experience

The questionnaire results revealed distinct patterns in cadets' perceptions of conventional learning and the PBL learning environment across the five measured dimensions. As presented in Table 6, PBL received higher mean scores for conceptual understanding, learning motivation, self-directed learning, and collaboration and communication. Conceptual understanding increased from 15.89 (SD = 3.38) under conventional learning to 16.51 (SD = 3.79) under PBL. Learning motivation showed the largest descriptive increase, from 14.83 (SD = 3.64) to 16.17 (SD = 4.23), consistent with previous meta-analytic findings on student motivation in PBL environments (Wijnia et al., 2024). Self-directed learning increased from 15.17 (SD = 3.44) to 15.94 (SD = 4.13), while collaboration and communication increased from 15.80 (SD = 3.46) to 16.38 (SD = 3.53).

In contrast, perceived problem-solving skills received a slightly lower mean score under PBL, decreasing from 15.80 (SD = 3.48) under conventional learning to 15.06 (SD = 4.72) under PBL. This slight decrease may reflect cadets' heightened awareness of cognitive barriers and task complexity when transitioning from procedural exercises to open-ended, real-world modeling tasks (Schukajlow et al., 2023). Thus, four of the five dimensions showed higher descriptive scores under PBL, whereas perceived problem-solving skills showed the opposite pattern. These findings suggest that cadets generally perceived the contextualized PBL experience positively in terms of conceptual understanding, motivation, self-directed learning, and collaboration and communication.

Table 6
Comparison of Cadets' Perceptions of Conventional Learning and PBL

Learning Dimension	Conventional Learning Mean ± SD	PBL Mean ± SD	Descriptive Comparison

Conceptual Understanding	15.89 ± 3.38	16.51 ± 3.79	Higher under PBL
Learning Motivation	14.83 ± 3.64	16.17 ± 4.23	Higher under PBL
Problem-Solving Skills	15.80 ± 3.48	15.06 ± 4.72	Lower under PBL
Self-Directed Learning	15.17 ± 3.44	15.94 ± 4.13	Higher under PBL
Collaboration & Communication	15.80 ± 3.46	16.38 ± 3.53	Higher under PBL

Note. Values are presented as mean ± standard deviation. Descriptive comparisons do not imply statistical significance.

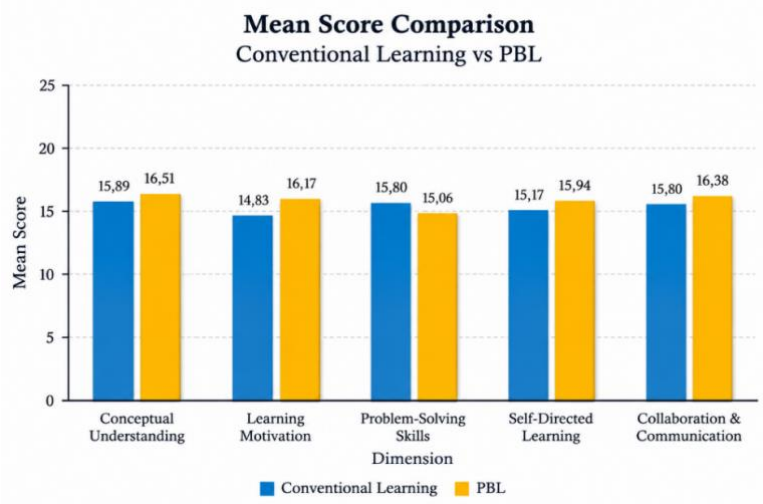


Figure 2. Comparison of Cadets' Mean Perception Scores Between Conventional Learning and PBL

Discussion

The Effect of Problem-Based Learning on Cadets' Learning Outcomes

The findings indicate that the implementation of Problem-Based Learning (PBL) was associated with a statistically significant improvement in cadets' learning outcomes in spherical trigonometry, consistent with previous evidence on problem-based mathematics instruction (Ayari et al., 2025). This improvement may be related to the core characteristics of the PBL intervention. Rather than learning spherical trigonometry primarily through formula-based or rote instruction, cadets were required to identify relevant mathematical information, organize geographical coordinates, select appropriate formulas, perform calculations, compare manual and digital results, and interpret their solutions within a real-world navigation context. Thus, the great-circle problems functioned not merely as routine computational exercises but as authentic tasks through which cadets actively constructed and applied mathematical knowledge (Sriantini & Abadi, 2024). This approach is consistent with the theoretical principles of PBL, which emphasize learning through meaningful problems, guided investigation, collaboration, and structured reflection (Dolmans et al., 2005).

The findings also support the role of contextualization in connecting abstract mathematical concepts with professional practice. By calculating distances between geographical positions and interpreting the results in terms of great-circle routes, cadets could directly relate spherical trigonometric procedures to a recognizable maritime application. Such contextualization increases the perceived relevance of mathematical content and provides opportunities for cadets to understand its professional value. This interpretation aligns with previous research suggesting that authentic, domain-oriented mathematics learning enhances conceptual understanding and mathematical reasoning through meaningful learning experiences (Suryanti et al., 2022). Nevertheless, the magnitude of improvement should be interpreted cautiously. The moderate Modified N-Gain and moderate-to-large effect size indicate that, although cadets' performance improved significantly, the intervention did not produce a

uniformly substantial improvement across all performance levels. Moreover, the one-group pretest–posttest design inherently limits causal claims because potential confounding factors, such as maturation, prior learning, testing effects, and historical events, cannot be completely ruled out. The findings therefore provide empirical evidence of learning improvement following a contextualized PBL intervention rather than definitive proof that PBL alone caused the improvement. Within these methodological boundaries, the results support the potential of contextualized PBL for teaching spherical trigonometry in nautical education.

Application of the Spherical Law of Cosines and Digital Verification

The findings for RQ2 indicate that cadets successfully applied the spherical law of cosines to calculate great-circle distances and effectively cross-checked their manual calculations against results generated by a web-based great-circle calculator. Across the three routes examined, the numerical differences were exceptionally small. For Harbor 1–Harbor 2, the mean difference was 0.16 km (approximately 0.006%), while for Harbor 1–Harbor 3, it was 0.08 km (approximately 0.001%). The largest difference occurred for Harbor 2–Harbor 3, at 4.06 km, representing approximately 0.049% of the digital reference distance. Inferential procedures provide further insight into the consistency between the two calculation methods. No statistically significant difference was found between manual and digital distances for Harbor 1–Harbor 2 ($Z = -0.327$, $p = .744$) or Harbor 1–Harbor 3 ($Z = -1.199$, $p = .230$), whereas a paired-samples t-test revealed a statistically significant difference for Harbor 2–Harbor 3 ($t(64) = -2.719$, $p = .008$). This result should be interpreted alongside the magnitude of the observed difference. Although statistically significant, the 4.06 km discrepancy over a distance of approximately 8,226 km represents a relative difference of only 0.049%, suggesting that its practical magnitude was small in the context of this computational comparison. Furthermore, the perfect linear correlation ($r = 1.000$) indicates that the manual and digital results exhibited identical linear variation across observations. However, correlation alone does not establish agreement between two measurement methods; the small absolute and relative differences provide additional evidence of their numerical consistency.

From a pedagogical perspective, this dual-verification task required cadets to evaluate navigation data through both manual trigonometric calculation and digital computation. The manual phase required cadets to transform geographical coordinates (latitude and longitude) into spherical trigonometric parameters, apply the spherical law of cosines Bowditch (2017), and interpret the resulting distances. The subsequent comparison with digital outputs enabled cadets to identify potential rounding discrepancies and verify mathematical consistency. Thus, the digital tool functioned as an analytical reference rather than a black-box replacement for mathematical reasoning (Baric et al., 2021). These findings are directly relevant to modern maritime operations. Although contemporary bridge technology provides rapid computational outputs, the ability to critically evaluate and verify digital data remains essential for safe navigation (Baric et al., 2021).

Integrating manual calculation with digital cross-checking provides a balanced learning context in which cadets connect fundamental mathematical principles with digital bridge operations. This approach enables them to use foundational mathematics to assess the plausibility and reliability of digitally generated navigation outputs rather than treating manual calculations and digital tools as competing modalities. These findings must nonetheless be interpreted within the scope of the study. High numerical consistency between manual and digital calculations demonstrates computational consistency but does not independently establish full shipboard operational competence, bridge resource management skills, or navigational decision-making under stressful conditions. Because cadets were not evaluated through full-mission simulator scenarios or live sea trials, the results demonstrate their ability to apply the spherical law of cosines and verify their calculations digitally within the instructional tasks, rather than proving complete operational readiness on the bridge.

Cadets' Perceptions of Contextualized Problem-Based Learning

The questionnaire findings offer detailed insights into cadets' perceptions of conventional instruction compared with the contextualized PBL environment. Compared with conventional learning, PBL yielded higher mean perception scores across four dimensions: conceptual

understanding (15.89 vs. 16.51), learning motivation (14.83 vs. 16.17), self-directed learning (15.17 vs. 15.94), and collaboration and communication (15.80 vs. 16.38). These descriptive trends indicate that cadets perceived the contextualized PBL environment more favorably than conventional lecture-based instruction (Mohd Rokeman, 2024). The increase in perceived conceptual understanding reflects the structural design of the intervention. Rather than completing abstract algebraic exercises, cadets connected geographical coordinates, spherical triangles, and great-circle routes directly to real-world voyages, transforming spherical trigonometry from an isolated mathematical exercise into a meaningful professional tool (Loyens et al., 2023).

Learning motivation showed the largest increase (+1.34 points), which may be attributed to the professional relevance of the navigation scenarios. Solving realistic voyage-planning problems involving major international ports provided an explicit professional purpose for mastering trigonometric formulas. This interpretation is consistent with extensive literature showing that authentic, problem-oriented learning environments can enhance student engagement and intrinsic motivation by highlighting the practical utility of learning content (Wijnia et al., 2024).

The higher rating for self-directed learning aligns with the inquiry-based nature of PBL, in which cadet groups had to independently identify variables, select mathematical models, and verify solutions with minimal direct instructor intervention while analyzing problems and formulating strategies (Savery, 2015). Similarly, the improvement in perceived collaboration and communication reflects the group dynamics incorporated into the intervention, which required cadets to articulate mathematical reasoning, deliberate over alternative approaches, and present their group findings to the class (Lachowsky & Murray, 2021).

In contrast, perceived problem-solving skills showed a slight decrease under PBL, falling from 15.80 (SD = 3.48) under conventional learning to 15.06 (SD = 4.72) under PBL. This finding should not be interpreted as evidence of a decline in actual problem-solving ability. One plausible interpretation is that authentic navigation problems introduced greater perceived task complexity and cognitive demands than routine procedural exercises. However, this interpretation remains tentative because cognitive load was not directly measured (Schukajlow et al., 2023). This distinction is crucial when comparing achievement data with perception scores. Although objective posttest scores improved significantly ($p < .001$), subjective ratings of problem-solving competence decreased slightly. Because the achievement test measured objective performance, whereas the questionnaire captured self-perceived competence, the two results address different constructs and should not be treated as directly contradictory.

Integration of Mathematical Reasoning and Digital Navigation Technologies

This study contributes to the broader academic discourse on integrating fundamental mathematical reasoning with digital navigation tools in maritime education. The intervention avoided framing manual mathematics and digital technology as opposing paradigms. Instead, the two were integrated sequentially, with cadets performing manual trigonometric calculations before conducting digital cross-checks. Pedagogically, this sequence creates a bridge between theoretical mathematics and modern technology. Manual computation helps ensure that cadets understand the geometric principles governing great-circle sailing, whereas digital cross-checking demonstrates how software automates these calculations. This combination helps cadets appreciate the efficiency of digital tools while maintaining the analytical capability required to identify software malfunctions, erroneous inputs, or system anomalies (Baric et al., 2021).

These results support the contextualized framework examined in this study, which integrates PBL pedagogy, spherical trigonometry grounded in great-circle navigation, and digital verification through manual calculation. This framework extends PBL into a specialized vocational domain and transforms mathematics from passive calculation into an active process of modeling, verifying, and communicating technical data. Importantly, these findings do not suggest that manual calculations should replace modern digital bridge systems such as Electronic Chart Display and Information Systems (ECDIS). Rather, they support a complementary relationship in which digital tools provide computational speed, while mathematical reasoning provides the

foundation for critical verification. This balance is vital in maritime training, where officers must possess both technological fluency and the mathematical competence needed to critically evaluate automated systems.

Taken together, the findings offer several practical implications for maritime mathematics education. First, spherical trigonometry should be taught contextually by embedding formulas such as the spherical law of cosines into authentic voyage-planning tasks (e.g., calculating great-circle distances and determining positions), thereby helping cadets appreciate the direct professional utility of mathematics (Sriantini & Abadi, 2024). Second, PBL offers an effective framework for integrating technical calculations, self-directed inquiry, and team collaboration, aligning mathematics instruction with contemporary outcome-based maritime training standards (Savery, 2015). Third, digital navigation tools should be introduced in mathematics classes as verification instruments rather than replacements for mathematical reasoning, thereby fostering habits of critical data evaluation among future navigational officers (Baric et al., 2021). Finally, the decrease in perceived problem-solving confidence highlights the need for structured instructional scaffolding, such as worked examples, guiding questions, and progressively challenging tasks, when transitioning cadets from procedural exercises to complex real-world problems (Lazonder & Harmsen, 2016).

Several methodological limitations should also be acknowledged. The study employed a one-group pretest–posttest design involving 65 cadets at a single institution. Without a control group, threats to internal validity, such as history and maturation, cannot be completely ruled out (Shadish & Cook, 2002). Future research should employ quasi-experimental non-equivalent control-group designs or randomized controlled trials across multiple institutions to strengthen internal and external validity. The sample was limited to nautical cadets studying spherical trigonometry at Politeknik Pelayaran Sorong; therefore, multicenter studies involving diverse maritime academies are recommended to evaluate the generalizability of these findings.

Student perceptions were collected through self-report questionnaires, which reflect subjective beliefs rather than direct measures of operational competence. Future studies should therefore incorporate objective performance rubrics, observational checklists, or full-mission simulator evaluations. Finally, digital verification in this study relied on a web-based great-circle calculator. Subsequent research should integrate full Electronic Chart Display and Information Systems (ECDIS) or bridge simulators to examine how cadets respond to digital system errors, sensor failures, and GPS spoofing through mathematical verification. The exact number and duration of the PBL instructional sessions were not retained in the available study records. This limitation affects procedural reproducibility and should be addressed by specifying these details prospectively in future replications.

In summary, this study provides empirical support for implementing contextualized Problem-Based Learning in spherical trigonometry instruction for nautical cadets. The statistically significant improvement in posttest scores, combined with a moderate Modified N-Gain and a moderate-to-large effect size, indicates meaningful academic gains following the intervention (Field, 2018). Meanwhile, the great-circle navigation task demonstrated that cadets successfully applied the spherical law of cosines, achieving high numerical consistency with digital reference outputs. Perception data complemented these achievement outcomes, showing that cadets evaluated PBL more favorably in terms of conceptual understanding, motivation, self-directed learning, and collaboration and communication. In contrast, the slight reduction in perceived problem-solving confidence highlights the increased cognitive complexity associated with authentic navigation problems rather than necessarily indicating a decline in actual problem-solving ability (Hänze & Leiss, 2022). Ultimately, integrating authentic great-circle navigation problems, manual trigonometric modeling, and digital cross-checking provides a promising framework for maritime mathematics education, enabling cadets to connect abstract spherical trigonometry with real-world bridge operations while fostering mathematical rigor, critical data verification, and collaborative problem-solving skills.

CONCLUSION

This study demonstrates that contextualized Problem-Based Learning (PBL), based on authentic great-circle navigation problems, can support the development of nautical cadets'

learning outcomes and navigation-related learning experiences. Learning outcomes increased from a pretest mean of 59.45 to a posttest mean of 71.32, with a statistically significant Wilcoxon signed-rank test result ($Z = -3.890$, $p < .001$), a Modified N-Gain of 0.340, and a moderate-to-large effect size ($r = 0.483$). These findings indicate meaningful improvement following the PBL intervention, although the one-group design does not permit the improvement to be attributed exclusively to PBL. Manual great-circle calculations also demonstrated high numerical consistency with digital navigation outputs, with percentage differences ranging from approximately 0.001% to 0.049% across the three routes. This supports the use of digital tools as verification mechanisms for manual calculations while reinforcing the importance of maintaining fundamental computational skills. However, computational agreement alone does not demonstrate full operational competence in bridge decision-making or real-world navigation.

Regarding cadets' perceptions, PBL was associated with higher ratings of conceptual understanding, learning motivation, self-directed learning, and collaboration and communication. In contrast, perceived problem-solving ability decreased, suggesting that increasingly complex navigation tasks may heighten cognitive demands and increase the need for appropriate instructional scaffolding. Therefore, PBL implementation should be supported by guiding questions, worked examples, and progressively challenging tasks.

Overall, the findings provide empirical support for contextualized PBL as a promising approach to integrating mathematical reasoning with authentic nautical navigation tasks. Future research should employ comparative or randomized research designs across multiple institutions and incorporate objective performance measures, simulator-based assessments, and ECDIS-based evaluations to determine whether these learning gains translate into operational navigation competence.

ACKNOWLEDGEMENT

The authors would like to thank Politeknik Pelayaran Sorong and the nautical cadets who participated in this study for their cooperation during data collection.

AUTHOR CONTRIBUTION STATEMENT

The first author contributed to the conceptualization and research methodology, data collection, data processing, statistical analysis, interpretation of the results, and preparation of the original manuscript. The second author contributed to data processing and statistical analysis, validation and interpretation of the research findings, literature review, and revision of the manuscript. The third author contributed primarily to language editing, grammar correction, English language refinement, proofreading, and improvement of the clarity, consistency, and readability of the manuscript. All authors reviewed and approved the final version of the manuscript and agreed to be accountable for the accuracy and integrity of the work.

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